Solutions

Tut1

(1) B’ ^ (A --> B) --> A’ = (B’ ^ (A’ v B))’ v A’ = (B’ ^ A’)’ v A’
= B v A v A’ = T

A	B	A’	B’	A --> B	B’ ^ (A --> B)	A’ ^ (A --> B) --> A’
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

(2) Prove (N’ ^ (F’ v N) ^ (A’ --> F)) --> A is a tautology

Let P = (N’ ^ (F’ v N) ^ (A’ --> F))

N	F	A	N’	N’ ^ (F’ v N)	A’ --> F	(N’ ^ (F’ v N) ^ (A’ --> F))	P --> A
T	T	T	F	F	T	F	T
T	T	F	F	F	T	F	T
T	F	T	F	F	T	F	T
T	F	F	F	F	F	F	T
F	T	T	T	F	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	F	F	T

(3) The question Q is Q = A^ B v A’ ^ B’? The following is the truth table for Q and the response from the asked guard.

A: The asked guard is honest

B: Door 1 is to freedom

A	B	Q	R
T	T	T	T
T	F	F	F
F	T	F	T
F	F	T	F

If answer is yes, go to exit 1

If the answer is no, go to exit 2

(4)

(for some x)x>0 ^ (for some x)x<0 is true

(for some x)(x>0 ^ x<0)) is false

(for all x)(for some y) x<y is true

(for some x)(for all y) x<y is false

(for all x) (x>3 --> x>2) is true
(for some x) x>3 --> (for all x) (x>2) is false

(for all x)(x>0)’ = (for all x) x • 0 is false
((for all x)x>0)’ = (for some x) x • 0 is true

Tut 2

(1) H(x) : x works hard. R(x) : x is rich.

```
    (for all x)(H(x) --> R(x))            (for some x)R’(x)
----------------------------------------  ------------------------------
          H(a) --> R(a)                          R’(a)
    ---------------------------------------------------------
                         H’(a)
             ---------------------------------
                  (for some x)H’(x)
```

(2) T(1) = 0
    T(2n) = 2T(n) + 2n-1
    Prove $T(n) = n\log_2 n – n + 1$

Basis k=1.
T(1) =0 by def. By theorem T(1) = 0- 1 + 1 = 0.
Inductive step. Suppose T(n) is true.

$T(2n) = 2T(n) + 2n - 1 = 2(n\log_2 n – n + 1) + 2n – 1$
$= 2n\log_2 n + 1$
$= 2n\log_2 n + 2n – 2n + 1$
$= 2n\log_2 2n – 2n + 1$

(3) F(0) and F(1) are straightforward.

Suppose F(n) is given by the formula.

$F(n+1) = (1/sqrt(5))((1+sqrt(5))/2)^n - (1/sqrt(5))((1 - sqrt(5))/2)^n$
$+ (1/sqrt(5))((1+sqrt(5))/2)^{n-1} - (1/sqrt(5))((1 - sqrt(5))/2)^{n -1}$
$= (1/sqrt(5))((1+sqrt(5))/2)^{n-1} ((1+sqrt(5))/2 +1)$
$- (1/sqrt(5))((1- sqrt(5))/2)^{n -1}((1-sqrt(5))/2 +1)$
$= (1/sqrt(5))((1+sqrt(5))/2)^{n-1} (3+sqrt(5))/2$
$-(1/sqrt(5))((1- sqrt(5))/2)^{n -1}(3-sqrt(5))/2$
$= (1/sqrt(5))((1+sqrt(5))/2)^{n-1} ((1+sqrt(5))/2)^2$
$- (1/sqrt(5))((1- sqrt(5))/2)^{n -1}((1-sqrt(5))/2)^2$
$= (1/sqrt(5))((1+sqrt(5))/2)^{n+1}$
$- (1/sqrt(5))((1- sqrt(5))/2)^{n+1}$

(4) C(n+1, k) = C(n, k) + C(n, k-1) = n!/((k!(n-k)!) – n!/((k-1)!(n-k+1)!)
    = n!/((k-1)!(n-k)!)(1/k + 1/(n-k+1))
    (n!/((k-1)!(n-k)!))((n+1)/((k(n-k+1))
    = (n+1)!/(k!(n+1-k)!)

Tut3

(1) S = {1, 2, …, 30}.
C = {3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25, 27, 30}
A = {3, 6, 9, …, 30}. B = {5, 10, 15, 20, 25, 30}. A • B = {15, 30}
|C| = 14 = |A| + |B| - |A • B| = 10 + 6 – 2 = 14

(2) |A U B U C| = |(A U B) U C| = |A U B| + |C| - |(A U B) • C|
= |A| + |B| - |A • B| + |C| - |(A • B) U (A • C)|
= |A| + |B| - |A • B| + |C| -(|A • C| + |B • C| - |A • C • B • C|
= A| + |B| - |A • B| + |C| -|A • C| - |B • C| + |A • C • B|

(3) Suppose theorem is true for n.

$|A_1 \cup A_2 \cup \ldots \cup A_n \cup A_{n+1}| = |A_1 \cup A_2 \cup \ldots \cup A_n| + |A_{n+1}|$
$- |(A_1 \cup A_2 \cup \ldots \cup A_n) \bullet A_{n+1}|$

$= |A_1| + |A_2| + \ldots + |A_n| - |A_1 \bullet A_2| - \ldots - |A_{n-1} \bullet A_n| + \ldots$
$+ \bullet (-1)^{k-1}|A_{i1} \bullet A_{i2} \bullet \ldots \bullet A_{ik}| + \ldots + (-1)^n|A_1 \bullet A_2 \bullet \ldots \bullet A_n|$
1 • i1 < … <ik • n
$+ |A_{n+1}|$
$- (|A_1 \bullet A_{n+1}| + |A_2 \bullet A_{n+1}| + \ldots + |A_n \bullet A_{n+1}| - |A_1 \bullet A_2 \bullet A_{n+1}| - \ldots - |A_{n-1} \bullet A_n \bullet A_{n+1}| + \ldots$
$+ \bullet (-1)^{k-1}|A_{i1} \bullet A_{i2} \bullet \ldots \bullet A_{ik} \bullet A_{n+1}| + \ldots + (-1)^n|A_1 \bullet A_2 \bullet \ldots \bullet A_n \bullet A_{n+1}|)$
1 • i1 < … <ik • n
$= |A_1| + |A_2| + \ldots + |A_{n+1}| - |A_1 \bullet A_2| - \ldots - |A_n \bullet A_{n+1}| + \ldots$
$+ \bullet (-1)^{k-1}|A_{i1} \bullet A_{i2} \bullet \ldots \bullet A_{ik}| + \ldots + (-1)^{n+1}|A_1 \bullet A_2 \bullet \ldots \bullet A_{n+1}|$
$1 \bullet i_1 < \ldots < i_k \bullet n+1$

(4) abc bac cab dab If you read column-wise from left to right
abd bad cad dac they are in lexico-graphic order
acb bca cba dba
acd bcd cbd dbc
adb bda cda dca
adc bdc cdb dcb

(5) abc Three items out of {a, b, c, d, e}
abd
abe
acd
ace
ade
bcd
bce
bde
cde

(6) aaabbb ababab aabbab abaabb aababb
((( ))), ( ) ( ) ( ), (( )) ( ), ( ) (( )), (( ) ( ))
(((( )))), ( ) ( ) ( ) ( ), (( ) ( )) ( ), ( ) (( ) ( )), …
(2n)!/(n!n!) – (2n)!/((n-1)!(n+1)!) = ((2n)!/(n!n!))(1 – n/(n+1)) = C(2n, n)/(n+1)