COSC222 Assignment 1 Discrete Structures

Question 1. The set of Fibonacci strings is the set of binary strings in which there is no occurrence of 11. A few examples are given. Null is the null string, the string of length 0.

length 0	length 1	length 2	length 3
null	0	0 0	0 0 0
	1	0 1	0 0 1
		1 0	0 1 0
			1 0 0
			1 0 1

These strings are arranged in lexico-graphic order, which is essentially increasing order when read as a binary number.

(1) Following this example, list up all Fibonacci strings of length 4 and 5 in lexico-graphic order.

(2) Let S(n) be the set of Fibonacci strings of length n. S(n) is recursively defined as

S(0) = {null}, S(1) = {0, 1}
S(n) = 0S(n-1) U 10S(n-2)

Here xS for string x and a set of strings S is the set of strings from S to which x is attached in the front, that is, S = {xy | y in S}. Note that y is taken in the order of elements in S. Confirm that the sets in (1) are given in this recursive manner. Let T(n) be the size of S(n). Obtain the solution for T(n), and confirm the value of T(4) by actually calculating the solution formula.

(3) The situation becomes similar if we forbid 00. Develop a similar theory as above, and obtain the corresponding S(4) and S(5). If you forbid 01 or 10, what are S(4) and S(5)?

Question 2. A multiset permutation is a permutation on a multiset. A multiset is a set where multiple occurrences of elements are allowed. For example, S={1, 2, 2, 3} is a multiset, where the number of occurrences of 2 is 2, and those for the rest are 1. The multiset permutations for S are given in lexico-graphic order as follows:

1 2 2 3	2 2 3 1	Read column-wise
1 2 3 2	2 3 1 2	
1 3 2 2	2 3 2 1	
2 1 2 3	3 1 2 2	
2 1 3 2	3 2 1 2	
2 2 1 3	3 2 2 1	

(1) Following this example, list up all permutations of S = {1, 1, 2, 2, 3}. Confirm the number of permutations by $n!/(n_1! \dots n_k!)$, where n_i is the number of occurrences of i, and k is the number of distinct elements. In the above example, k=3.

(2) Multiset combinations can be defined similarly. For example, the set of 3-multiset combinations for S = {1, 1, 2, 2, 3} is give in lexico-graphic order as follows:

1 1 2 Three items are taken out of S.
1 1 3 Each combination is given in non-decreasing order of integers.
1 2 2
1 2 3
2 2 3

Following this example, list up all 4-multiset combinations for S={1, 1, 2, 2, 2, 3, 3} in lexico-graphic order.

Question 3. Binary adder
(1) Draw the logic network of binary adder with four inputs.
(2) Expand the last two digits of your student number in binary, and take the last four bits for input_1. If you have all 0, add 1. Trace the logic network obtained in (1) with input_1 and input_2 = (1 1 0 1). For trace attach 0 or 1 at each gate, input and output. Confirm the output is correct with your hand calculated result.

Question 4 Logic network

(1) Obtain the minimum logic formula for the following logic function by the Karnaugh map and Quine-McCluskey procedure.

x_1	x_2	x_3	x_4	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

(2) Draw a logic network for the function f. Try to minimize the number of logic components.

Due date : 3 April 2008, worth 20%, drop due one week after.