

COSC222 Supplementary Note: Inference Rules for Propositional Logic

$P, P \rightarrow Q$		$P \rightarrow Q, Q'$	
Q	modus ponens	P'	modus tollens
P, Q		$P \wedge Q$	
$P \wedge Q$	conjunction	P, Q	simplification
P		$P \rightarrow Q, Q \rightarrow R$	
$P \vee Q$	addition	$P \rightarrow R$	hypothetical syllogism
$P' \vee Q$		$(P \wedge Q)'$	
$P \rightarrow Q$	implication	$P' \vee Q'$	de Morgan

Example Socrates is a man, man \rightarrow dies (This needs predicate logic)
----- modus ponens
Socrates dies

Example Prove $((A' \wedge B) \wedge (B \rightarrow C)) \rightarrow (A \rightarrow C)$

$A' \vee B$	
$A \rightarrow B$ $B \rightarrow C$	
$A \rightarrow C$	

This is called a **proof chart**

Example. (R) Russia was a superior power. (F) France was strong.
 (N) Napoleon made an error. (A) Army failed.

Prove that:

Russia was a superior power, and either France was not strong or Napoleon made an error. Napoleon did not make an error, but if the army did not fail, then France was strong. Hence the army failed and Russia was a superior power.

$F' \vee N, R$	
$N', F \rightarrow N, A' \rightarrow F$	
$N', A' \rightarrow N$	hypothetical syllogism
A, R	modus tollens

R is carried all the way down, being omitted in the intermediate stages.

Direct proof of $((A' \wedge B) \wedge (B \Rightarrow C)) \Rightarrow (A \Rightarrow C)$

0 and 1 correspond to F and T.

Build up truth tables form smaller components to larger components.

Use the fact that the truth value of $X \Rightarrow Y$ is 0 only for $X=1$ and $Y=0$. Otherwise 1.

A	B	C	$A' \wedge B$	$B \Rightarrow C$	$(A' \wedge B) \wedge (B \Rightarrow C)$	$A \Rightarrow C$	$((A' \wedge B) \wedge (B \Rightarrow C)) \Rightarrow (A \Rightarrow C)$
0	0	0	0	1	0	1	1
0	0	1	0	1	0	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	0	0	0	0	1
1	1	1	0	1	0	1	1

Exercise. Prove $((A \Rightarrow B) \wedge (C' \vee A) \wedge C) \Rightarrow B$ by inference rules or truth table.

Interpretation

A : You work hard.

B : Degree is awarded.

C : Your condition is good.

If you work hard, degree will be awarded. Either your condition is bad or you work hard. Your condition is good. Therefore degree is awarded.

Recurrence equations and recursive algorithms

An example of a recurrence equation is

$$T(1)=1$$

$$T(n-1) = T(n-1) + 2n - 1 \quad (n \geq 2)$$

Generally $T(n)$ is defined in terms of the previous values of $T(n)$.

Exercise. Prove that $T(n) = n^2$ by induction.

This value of $T(n)$ can be computed by the following recursive C program.

```
int n;
int T(int n){
    if (n==1) return 1;
    else return T(n-1) + 2*n - ;
}
main(){ scanf("%d", &n); printf("%d\n", T(n)); }
```