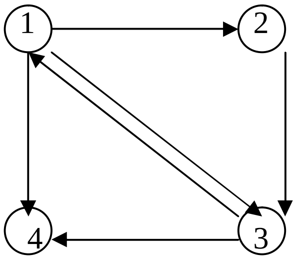
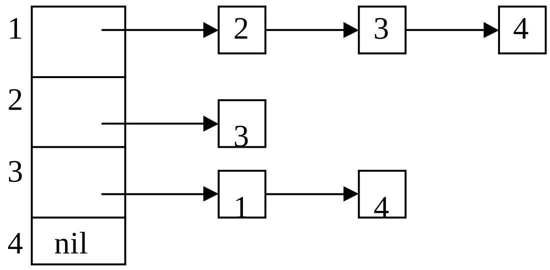
Solutions for tutorial 6

(1)





(2) $A = \begin{matrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$ $A^2 = \begin{matrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$ $A^3 = \begin{matrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$

$R = \begin{matrix} 3 & 1 & 3 & 4 \\ 1 & 2 & 2 & 2 \\ 2 & 1 & 3 & 3 \\ 0 & 0 & 0 & 1 \end{matrix}$ $R^* = \begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{matrix}$

(3) 1 • 4
1 • 3 • 4
1 • 2- 3 • 4, 1 • 3 • 1 • 4
1 • 2 • 3 • 1 • 4, 1 • 3 • 1 • 3 • 4
1 • 3 • 1 • 3 • 1 • 4, 1 • 3 • 1 • 2 • 3 • 4 , 1 • 2 • 3 • 1 • 3 • 4
1 • 2 • 3 • 1 • 2 • 3 • 4 , 1 • 3 • 1 • 3 • 1 • 3 • 4 , 1 • 2 • 3 • 1 • 3 • 1 • 4
1 • 3 • 1 • 2 • 3 • 1 • 4

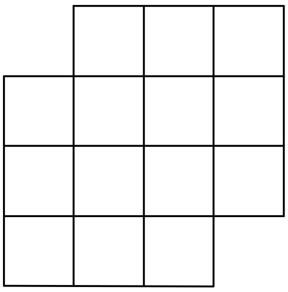
(4) Proof of 3f <= 2e. Let b_i be the number of edges surrounding face i. Since each edge is the boundary of two faces, b_1 + €   + b_f = 2e. Since each face is surrounded by at least 3 edges, b_i >= 3. Thus 3f <= b_1 + €   + b_f = 2e. Similarly, if each face is surrounded by at least four edges, we have 4f <= 2e.

Now we prove $K_{3,3}$ is non-planar. Since the minimum cycle of $K_{3,3}$ has at least four edges, we have 4f <=2e. Together with Euler•s theorem f+n• e = 2, we have e <= 2n • 4. In K3,3, we have n=6 and e=9, which does not satisfy the above inequality.

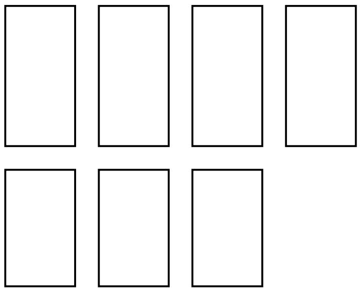
(5) Paint the rooms in the floor black and white like a checker board with the reception room white. Suppose there is a Hamilton path. Then the path from the reception will be like white-black-white-black-€   There are 24 rooms in the sequence, of which 13 are white and 11 are black, which is impossible.

This proof technique belongs to the wide definition of parity theory. The essence is that if there is a solution, it must satisfy a parity or near parity in some quantity.

Exercise. Fill the following room by the seven tiles.



Room with 14 units



7 tiles with 2 units each