COSC222 Tutorial Propositional and Predicate Logic

(1) The tautology of modus ponens can be proved in the following two ways.

Modus ponens is rewritten in the form of implication \( A \land (A \implies B) \implies B \).

Logical transformation. AND is simplified by concatenation
\[
A \land (A \implies B) \implies B = (A(A' \lor B))' \lor B = (AB)' \lor B
= A' \lor B' \lor B = T
\]

Truth table

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A \implies B</th>
<th>A \land (A \implies B)</th>
<th>A \land (A \implies B) \implies B</th>
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Following this example, prove the tautology of modus tollens.

(2) Prove the tautology of \((N' \land (F' \lor N) \land (A' \implies F)) \implies A\) by truth table. The “F” here is not the truth value false.

(3) A logician was captured by a mafia group, detained in a cell room. There were two guards beside the room. One is honest and the other is dishonest. There are two exits from the room, one to freedom and the other to death. The chief of the group said to him, “You can ask a guard a question only once, and go through either exit depending on the answer”. After a while the logician worked out a question, and went to freedom. What is the question?

Note. The dishonest guard replies the opposite truth value to a question of a logical formula, whereas the honest respond the truth value itself. The logician cannot know which guard he is asking.

Hint. Proposition A : The asked guard is honest
Proposition B : Door 1 is to freedom
A simple question like \(A'\)? (Are you dishonest?), or \(B\)? (Is door 1 to freedom?) does not help. The question must be a composite one, such as \(A \land B'\). This does not work. Why?

(4) Give interpretation to prove each of the following statements is not valid.
\[
\begin{align*}
(\text{for some } x)A(x) \land (\text{for some } x)B(x) & \not\implies (\text{for some } x)(A(x) \land B(x)) \\
(\text{for all } x)(\text{for some } y)P(x, y) & \not\implies (\text{for some } x)(\text{for all } y)P(x, y) \\
(\text{for all } x)(P(x) \land Q(x)) & \not\implies (\text{for some } x)P(x) \land (\text{for all } x)Q(x) \\
(\text{for all } x)(A(x))' & \not\implies (\text{for all } x)A(x)' 
\end{align*}
\]