

COSC222 Assignment 2 Discrete Structures

Question 1. A vending machine has two buttons to sell coke and juice with button 1 and button 2, and may be out of order. That is, button 1 sells coke and button 2 sells juice (normal), or button 1 sells juice and button 2 sells coke (out of order). The function is limited to (C, J) or (J, C), not (C, C) or (J, J) for (button1, button 2) even in condition out of order. If we try one button, we can tell which button is for which beverage. Let $b(i)$ be the output for button i for $i=1, 2$. We want to conclude

$$(b(1)=C \rightarrow b(2)=J) \wedge (b(1)=J \rightarrow b(2)=C)$$

The proof chart is given below. Note we put tautology $(b(1)=C \vee b(1) \neq C)$.

$$(b(1)=C \vee b(1) \neq C) \wedge (b(1)=C \wedge b(2)=J \vee b(1)=J \wedge b(2)=C), (b(1)=C \wedge b(1) \neq C) = F$$

$$(b(1)=C \wedge b(2)=J) \vee (b(1) \neq C) \wedge (b(1)=J \wedge b(2)=C)$$

$$b(1) \neq C \vee b(2) = J$$

$$b(1) = C \rightarrow b(2) = J$$

Similarly we can derive $b(1)=J \rightarrow b(2) = C$ by putting tautology $(b(1)=J \vee b(1) \neq J)$.

Now we have another vending machine with three buttons (1, 2, 3), whose function is intended to be (C, J, S). Here S means a surprise, that is, it gives coke or juice with 50% of chance each. The machine is completely out of order, meaning no button works properly. Hint. The function is (J, S, C) or ((S, C, J).

(1) Show testing one button is enough to tell which function the machine offers. Give the reason informally. Testing button 1 does not work. Why?

(2) Following the example given above, give a formal proof to (1) including the hint.

Prove $(b(1)=J \wedge b(2)=S) \wedge b(3)=C) \vee (b(1)=S \wedge b(2)=C \wedge b(3)=J)$, and then

$$(b(3)=J \rightarrow b(1)=S \wedge b(2)=C) \wedge (b(3)=C \rightarrow b(1)=J \wedge b(2)=S)$$

Question 2. A multiset permutation is a permutation on a multiset. A multiset is a set where multiple occurrences of elements are allowed. For example, $S=\{1, 2, 2, 3\}$ is a multiset, where the number of occurrences of 2 is 2, and those for the rest are 1. The multiset permutations for S are given in lexico-graphic order as follows:

1 2 2 3	2 2 3 1	Read column-wise
1 2 3 2	2 3 1 2	
1 3 2 2	2 3 2 1	
2 1 2 3	3 1 2 2	
2 1 3 2	3 2 1 2	
2 2 1 3	3 2 2 1	

(1) Following this example, list up all permutations of $S = \{1, 1, 2, 3, 3\}$. Confirm the number of permutations by $n!/(n_1! \dots n_k!)$, where n_i is the number of occurrences of i , and k is the number of distinct elements. In the above example, $k=3$.

(2) Multiset combinations can be defined similarly. For example, the set of 3-multiset combinations for $S = \{1, 1, 2, 3, 3\}$ is given in lexico-graphic order as follows:

1 1 2 Three items are taken out of S.
 1 1 3 Each combination is given in non-decreasing order of integers.
 1 2 3
 1 3 3
 2 3 3

Following this example, list up all 4-multiset combinations for $S = \{1, 1, 2, 2, 3, 3, 3\}$ in lexico-graphic order.

Question 3. Binary adder

- (1) Draw the logic network of binary adder with four inputs.
- (2) Expand the last two digits of your student number in binary, and take the last four bits for input_1. If you have all 0, add 1. Trace the logic network obtained in (1) with input_1 and input_2 = (1 0 1 1). For trace attach 0 or 1 at each gate, input and output. Confirm the output is correct with your hand calculated result.

Question 4 Logic network

- (1) Obtain the minimum logic formula for the following logic function by the Karnaugh map and Quine-McCluskey procedure.

x_1	x_2	x_3	x_4	f

0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

- (2) Draw a logic network for the function f. Try to minimize the number of logic components.

Due date : 2 June 2009, worth 20%, drop due one week after.

Marking policy Q1 (1) 5 (2) 5 , Q2 (1) 5 (2) 5, Q3 (1) 5 (2) 5, Q4 (1) 5 (2) 5

Total 40 marks