Post's Correspondence Problem

Post's correspondence problem (PCP)

Let A={w1, w2, …, wn} and B={x1, x2, …, xn} be two lists of strings over a finite alphabet. PCP ha a solution if there is an index set (i1, …, ik) such that

$$w(i1)w(i2)\ldots w(ik) = x(i1)x(i2)\ldots x(in)$$

Theorem. PCP is unsolvable.

Proof. If PCP is solvable, the Halting problem of Turing machines must be solvable, which is a contradiction.

Example.

	List A	List B
i	w(i)	x(i)
1	1	111
2	10111	10
3	10	0

Solution (2, 1, 1, 3) string 101111110

w2w1w1w3 = (10111)((1)(1)(10)

x2x1x1x3 = (10)(111)(111)(0)

Example

	List A	List B
i	w(i)	x(i)
1	10	101
2	011	11
3	101	011

This does not have a solution.

A: 10

B: 101

There are two strings from A that star from 1. Choice of index 1 brings

A: 1010

B: 101101

Choice of index 3 brings

A: 10101

B: 101011

This is the first situation, and this process goes forever.

From the unsolvability of PCP, we can show that of ambiguity of CFG.