COSC222 Tutorials for Context-Free Languages

Question 1

The following is a PDA (non-deterministic) that accepts the language L = $\{ww^R \mid w \in \{a, b\}^*\}$, where w^R is the reversal of w. That is, L is the set of palindromes of even length 2 and up. The following PDA accepts L by empty stack, where e is the empty string.

(state, stack top) \ input	a	b
(q0, e)	(q0, a)	(q0, b)
(q0, a)	(q0, aa), (q1, e)	(q0, ba)
(q0, b)	(q0, ab)	(q0, bb), (q1, e)
(q1, a)	(q1, e)	
(q1, b)		(q1, e)

The meaning of transition (q, α) ➔ (p, β) under input a is that when the automaton is in state q, and the input symbol is a, the string α at the top of stack is replaced by string β.

Two entries in the table give a non-deterministic choice. Possible unsuccessful and successful sequences of trace are:

```
        start          a               b                 b              b
(q0, abbbba, e) ➔ (q0, bbbba, a) ➔ (q0, bbba, ba) ➔ (q1, bba, a)  ➔ stop

        start       a                  b                 b
(q0, abbbba, e) ➔ (q0, bbbba, a) ➔ (q0, bbba, ba) ➔ (q0, bba, bba)
   b              a             a
➔ (q1, ba, ba) ➔ (q1, a, ,a) ➔ (q1, e, e)   accept
```

The meaning of (q, w, α) in the traces is that the current state is q, the remaining input string is w and the string in the stack is α. An input symbol that triggered the transition is attached above each ➔.

(1)Following this example, design a PDA for accepting odd palindromes by empty stack.

(2) Trace it with aabaa for unsuccessful trace and successful trace.

Question 2. L = $\{a^n b^n c^n \mid n \geq 1\}$ can be generated by

S → ASBC | ABC
CB → BC
AB → ab
Aa → aa
bB → bb
bC → bc
cC → cc

Sample derivation for aaabbbccc

S ➔ ASBC ➔ AASBCBC ➔ AAABCBCBC ➔ AAABCBBCC
➔ AAABBCBCC ➔ AAABBBCCC ➔ AAabBBCCc ➔ AaabBBCCC
➔ aaabBBCCC ➔ aaabbBCCC ➔ aaabbbCCC ➔aaabbbcCC ➔ aaabbbccC
➔ aaabbbccc

In this derivation the last c is generated last.
(1) Following this example, design a CSG for generating L with the first rule given by S → ABSC and the first a is generated last.
(2) Trace the generation for the same string using your grammar

Note. Pumping Lemma for CFL
If a CFL is infinite, there is w such that w=xyzuv such that |yu|>0 and
for any i, xy^izu^iv is in L.
Using this lemma, we can show the above L to be non-context-free.
Proof. Suppose L is context-free. Then try to divide a large w into xyzuv.
We face a contradiction in every case.

|aa …. a |bb … b| cc … c | |
- -------------------------------------
| x | y | z | u | v |

Question 3. Let L = {xx | x ∈ {a, b}*}. Prove L is non-context-free.
Hint. Consider the intersection of L and a*b*a*b*. The intersection of a context-free language and regular language is context-free.