

COSC222 Tutorial on Context-free Grammars

Question 1. Let G be defined by

$$S \rightarrow SS, S \rightarrow aSb, S \rightarrow bSa, S \rightarrow ab, S \rightarrow ba$$

Let L be defined by $L = \{x \mid \text{equal numbers of a's and b's occur in } x\}$.

- (1) Prove that $L(G) = L$. Hint Show both $L(G) \subseteq L$, and $L(G) \supseteq L$.
- (2) Draw all derivation trees for $ababbaba$
- (3) Obtain a Chomsky normal form for G
- (4) Trace the CYK algorithm for the above string for one derivation tree.

Question 2. Let G be defined by

$$S \rightarrow SS, S \rightarrow (S), S \rightarrow ()$$

Let L be defined by $L = \{x \mid x \text{ is a balanced string of "("s and ")"s}\}$

- (1) Prove that $L(G) = L$
- (2) Draw all derivation trees for $((())()())$
- (3) Same as for Question 1.
- (4) Same as for Question 1

Question 3. Let G be given by

$$S \rightarrow aSb \mid aS \mid e, \text{ where } e \text{ is the null string}$$

- (1) Describe $L(G)$.
- (2) Draw all derivation trees for $aaaabb$.
- (3) Obtain the formula for the number of derivation trees for $\{a^m b^n \mid m \geq n\}$, and confirm the number in (2).
- (4) Prove that $L(G)$ is not regular. Hint. If L is regular so is its reversal. If L_1 and L_2 are regular, so is $L_1 \cap L_2$.

Pumping lemma. If L is infinite and regular, there is w in L such that $w=xyz$, y is not empty and xy^iz is in L for all $i=1, 2, 3, \dots$

Tutorial on $\{a^n b^n \mid n \geq 1\}$ is not regular.

If y is all a , or all b , we lose the balance of a 's and b 's. If y include a and b , xy^2z is not in L .