COSC222 Tutorial on Context-free Grammars

Question 1. Let G be defined by

\[ S \rightarrow SS, \quad S \rightarrow aSb, \quad S \rightarrow bSa, \quad S \rightarrow ab, \quad S \rightarrow ba \]

Let L be defined by \( L = \{ x \mid \text{equal numbers of a’s and b’s occur in } x \} \).

1) Prove that \( L(G) = L \). Hint: Show both \( L(G) \subseteq L \), and \( L(G) \supseteq L \).

2) Draw all derivation trees for ababababa

3) Obtain a Chomsky normal form for G

4) Trace the CYK algorithm for the above string for one derivation tree.

Question 2. Let G be defined by

\[ S \rightarrow SS, \quad S \rightarrow (S), \quad S \rightarrow ( ) \]

Let L be defined by \( L = \{ x \mid x \text{ is a balanced string of } "(\)"\} \).

1) Prove that \( L(G) = L \)

2) Draw all derivation trees for \((())())()

3) Same as for Question 1.

4) Same as for Question 1

Question 3. Let G be given by

\[ S \rightarrow aSb \mid aS \mid e, \text{ where e is the null string} \]

1) Describe L(G).

2) Draw all derivation trees for aaaaabb.

3) Obtain the formula for the number of derivation trees for \( \{a^m b^n \mid m \geq n\} \), and confirm the number in (2).

4) Prove that L(G) is not regular. Hint: If L is regular so is its reversal. If \( L_1 \) and \( L_2 \) are regular, so is \( L_1 \cap L_2 \).

Pumping lemma. If L is infinite and regular, there is w in L such that w=xyz, y is not empty and xy^iz is in L for all \( i=1, 2, 3, \ldots \)

Tutorial on \( \{a^n b^n \mid n \geq 1\} \) is not regular.

If y is all a, or all b, we lose the balance of a’s and b’s. If y include and b, \( xy^2z \) is not in L.