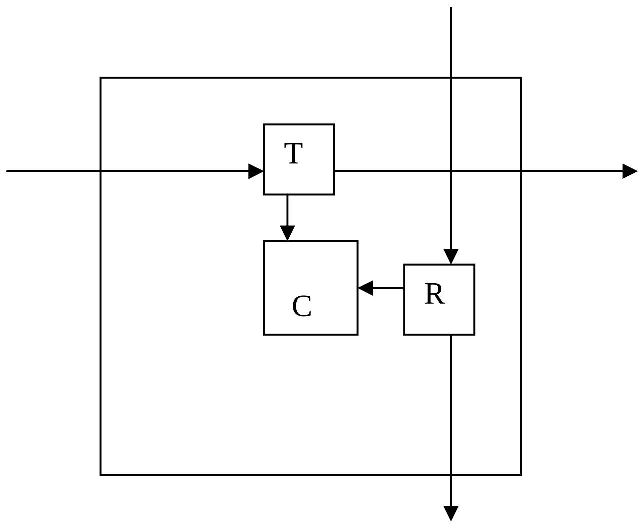
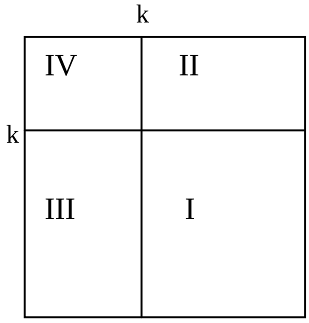
Skewed Matrix Method and Mesh Floyd

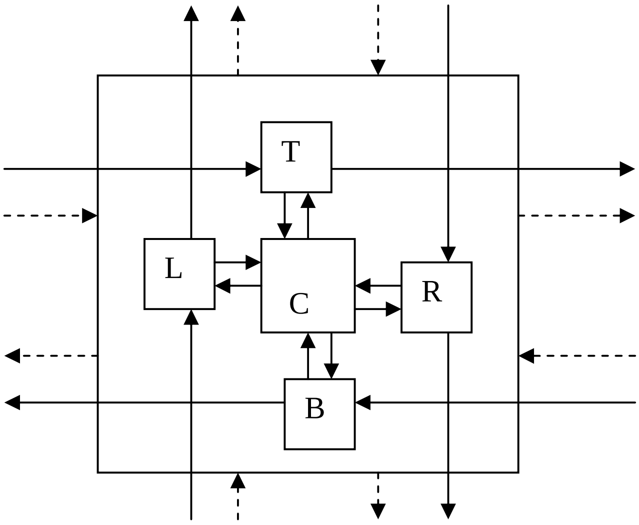
One cell of Skewed Matrix Method is given below. T (top register holds aik and R (right register) holds bkj. C (center register) holds $s=a_{i1}b_{1j} + \dots + a_{i,k-1}b_{k-1,j}$. $s = s + a_{ik}b_{kj}$ is carried out when a_{ik} and b_{kj} are ready. At the end C will hold $\sum_{[k=1,n]} a_{ik}b_{kj}$. This is a general description of matrix multiplication on a semiring. For a distance semiring + is interpreted as min and · is regarded as +. Thus C takes the value of $\min\{s, a_{ik}+b_{kj}\}$.
By skewing A and B, data items are synchronized to result in correct values in the product of A and B.



Floyd's algorithm for computing A* (We assume all diagonal are 0 for distance semiring)
  for k=1 to n do
      for i=1 to n do for j=1 to n do $a_{ij} = \min\{a_{ij} , a_{ik} + a_{kj}\}$

A cell of Mesh Floyd is given below. T (top register) or B (bottom register) hold a_{ik} and R (right register) or L (left register) hold a_{kj}. C (center register) holds current a_{ij}.
$C = C + a_{ik}a_{kj}$ is carried out in C when a_{ik} and a_{kj} are ready for general semiring. $C=\min\{C, a_{ik}+a_{kj}\}$ for a distance semiring. We call this action "update". For distance semiring, + is min and · is +. When a control signal comes horizontally the value in C is copied to L and R, and when it comes vertically, C is copied to T and B. Control signals propagate further after spending one unit of time at each cell. Control signal 1, …, n originate at cell(k, k) for each k at the interval of three steps one by one, and spread to four directions. Control signal k originates at time 3(k-1). Update is carried out using (T, R), (L, T), (R, B), and (B, L) for regions I, II, III, and IV





Solid arrows for data and dotted arrows for control signals.

Timing function for update by k-th control signal at cell(i, j)
We count from 0 for i, j, k.

$$T(i, j, k) = 3k + |i - k| + |j - k|$$

Then the last update takes place at (0, 0) with k = n-1, which is

$$T(0, 0, n-1) = 3(n - 1) + n - 1 - 0 + n - 1 - 0 = 5n - 5.$$

Alternatively, if we count from 1, T(i, j, k) is given by

$$T(i, j, k) = 3(k-1) + |i - k| + |j - k|$$

The last update with k=n takes place at (1, 1) at time

$$T(1, 1, n) = 3(n-1) + n - 1 + n - 1 = 5n - 5$$

The verification of timing in the note can apply to both interpretations.