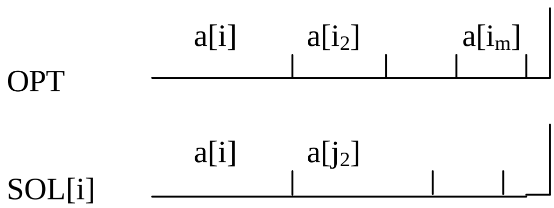
Exam2008 Solutions

(5.4) Suppose the optimal solution is $a[i_1]+ \ldots + a[i_m]$ for some m, and $(a[i_1], \ldots , a[i_m])$ is sorted. By Smart Algorithm, we must have the occasion that $i=i_1$. The value of sol must be no less than the value of s computed for $i=i_1$, which we call sol[i]. If there is only one item in the optimal solution, Smart Algorithm will catch this. If there are only two items in opt, Smart Algorithm will catch a[i] and $a[i_2]$ by sweeping by j. Thus we assume there are three or more items in the optimal solution.

Let the items packed for sol[i] be $(a[i], a[j_2], \ldots, a[j_k])$ for some k, that is, sol[i]= $a[i]+a[j_2]+\ldots+a[j_k]$. This indexing is for the convenience for the proof. We can regard $i=i_1$. Now we prove opt-sol[i] $\leq a[i_m]$. Observe that opt and sol[i] share the same first item. From the second item onwards, we have $a[i_2] \leq a[j_2]$, $a[i_3] \leq a[j_3]$, …, since items in opt are sorted, and items in sol[i] are sorted from the second item onwards. See the following picture.



Let $a[j_{k+1}]$ be the item that failed to be packed at the last trial by j. Then the range for $a[j_{k+1}]$ starts from some point in the range of $a[i_m]$ in opt. Otherwise we can pack $a[i_m]$ or smaller item for $a[j_{k+1}]$ by Smart Algorithm, a contradiction. Note that $a[i_m] \leq opt/3$, as those items are sorted in non-increasing order. Thus we have proven opt-sol[i] $\leq a[i_m] \leq$ opt/3. That is, sol[i] is greater than or equal to (2/3)opt.

(5.5) Let us force to pack up to K items. Similar argument can prove that opt - sol(I) $\leq a[i_m] \leq$ opt/(K+2) for some index set I. From this sol(I) is greater than or equal to opt(1 – 1/(K+2)).

(3.1) $x(n) = (2+sqrt(3))^n + (2-sqrt(3))^n$.

(3.3) Prove that $x(2n) = x(n)^2 - 2$.

$$x(n)^2 = ((2+sqrt(3))^n + (2-sqrt(3))^n)^2 = (2+sqrt(3))^{2n} + (2-sqrt(3))^{2n} + 2(4-3)^n$$
$$= x(2n) + 2$$

(4.3) For a multiple of 4, m, compute

$$(m/2-1)(m/2-1) = m^2/4 - m + 1 = m(m/4 - 1) + 1.$$

Since m/4 – 1 is an integer, we have (m/2-1)(m/2-1) mod m = 1. Similar for (m/2+1)(m/2+1).