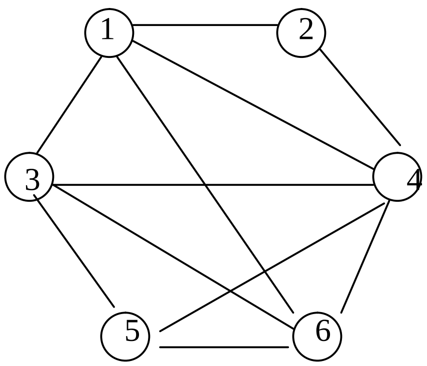
COSC 413 Written Assignment, due October 2, 2009, 5pm

Question 1. Transform the following CNF SAT problem into a 3-clique problem and discuss all corresponding solutions.

F = (A + B• + C•)(A• + B• + C)(A• + B + C),
X• is complement of X

Question 2. Transform the following 4-clique problem into a 2-vertex cover problem and discuss all corresponding solutions.

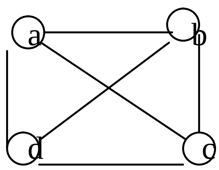


Question 3. Transform one of the vertex cover problem obtained above into a feedback edge set problem and discuss all corresponding solutions.

Question 4. transform the above vertex cover problem into a directed Hamilton circuit problem and discuss corresponding solutions.

Question 5. Transform the 3-SAT in question 1 into the corresponding colorability problem, and discuss corresponding two solutions.

Question 6. Transform the following colorability problem into the corresponding exact cover problem, and discuss two corresponding solutions.



Question 7. Transform the following exact cover problem into the corresponding knapsack problem, and discuss all corresponding solutions.

S1 = {1, 2, 3}, S2 = {1, 2, 4}, S3 = {2, 3, 4}, S4 = {4, 5}, S5={3, 5}

Question 8.
(1) Construct the multiplication table of Z_{15}*
(2) Find as many subgroups as possible.
(3) Obtains the right cosets for each subgroup
(4) Confirm Fermat•s Theorem a^{12} mod 13 = 1 with repeated squaring.
Specifically show a^2 à a^2a à $(a^3)^2$ à $((a^3)^2)^2$.for 3 different a.

Question 9. Let a difference equation be defined by

x(0) = 2, x(1) = 4
x(n+2) = 4x(n+1) - x(n) (n=0, 1, ...)

(9.1) Obtain the analytical solution for this equation.

(9.2) Transform the equation into a vector-matrix form as follows:

x(n+1) = 4x(n) - x(n-1)

This is transformed to the following.

(x(1), x(0)) = (1, 0)

$$(x(n+1), x(n)) = (x(n), x(n-1)) \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = (x(n), x(n-1))\ A$$

Obtain the matrix A. Then we have (x(n+1), x(n)) = (x(1), x(0)) A^n. Obtain x(6) by using the analytical solution, repeated use of the equation (3.2) five times and the repeated squaring of the form $(A^2A)^2$, and confirm that you have the same results.

(9. 3) Prove that $x_{2n} = x_n^2$ -2.

Question 10. Let n positive integers a[1], € , a[n] and a positive integer b be given. We consider the following knapsack problem.

max { • a[i]}, subject to • a[i] , b
I i in I i in I

Example. a = (6, 12, 4, 9, 15), b=35. Optimal solution is 34 = 6 + 4 + 9 + 15 with I={1, 3, 4, 5}, or (1, 0, 1, 1, 1) in binary vector.

We develop the following two approximation algorithms.

```
Greedy Algorithm  // In the following rem is the remaining capacity of the knapsack
sort a;           // array a is sorted in non-increasing order
sol=0;  rem=b;    // At the end, sol is the solution by this algorithm
for i=1 to n do begin
  if a[i] ,  rem then begin sol=sol+a[i]; rem=rem-a[i] end
                 else return sol
end.
```

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Smart Algorithm
sol=0;  // At the end, sol is the solution by this algorithm
for i=1 to n do begin
    s=a[i]; rem=b-a[i]; (*)
    for j=1 to n do
        if i ƒ j and a[j] ,  rem then begin s=s+a[j]; rem=rem „ a[j] end;
    sol = max{s, sol};
end
return sol.
```

The idea of Greedy Algorithm is to pack items from large to small one by one until no more is packed. The idea of Smart Algorithm is that for each item i, we force item i to be packed at line (*), and go greedy for the remaining capacity. Then we take the best solution from such n attempts.

(10.1) Trace Greedy Algorithm with the above example.

(10.2) Trace Smart Algorithm with the above example.

(10.3) Prove that the solution sol of Greedy Algorithm satisfies sol …opt/2, where opt is the optimal solution..

(10.4) We prove that the solution of Smart Algorithm satisfies sol …{2/3}*opt Complete the following description of the proof.

Suppose the optimal solution is $a[j_1]$+ €   + $a[j_m]$ for some m, and ($a[j_1]$, €   , $a[j_m]$) is sorted. By Smart Algorithm, we must have the occasion that $i=j_1$. The value of sol must be no more than the value of s computed for $i=j_1$.

(10.5) If we force k items are packed, and follow the greedy approach for the remaining capacity, and the best value over all combinations of k items, what approximation ratio can we achieve? Prove the ratio.