(1) Convert the following satisfiability problem into the corresponding clique problem, and discuss all corresponding solutions.

\[ F = (x_1 + x_2)(x_1 + x_2' + x_3')(x_1' + x_3') \]

Note. \( x' \) is the negation of \( x \), multiplication is ‘and’, and addition is ‘or’.

(2) Convert the following vertex cover problem into the corresponding feedback edge set problem and discuss corresponding solutions.

![Graph](attachment:graph.png)

Number theory

(3) Let a difference equation be defined by

\[ x(0) = 0, \quad x(1) = 1 \]
\[ x(n+2) = 2x(n+1) + x(n) \quad (n=0, 1, ...) \]

(3.1) Obtain the solution for this equation.

(3.2) Transform the equation into a vector-matrix form as follows:

\[
\begin{pmatrix}
  x(1) \\
  x(0)
\end{pmatrix}
= \begin{pmatrix} 0 & 1 \\ x(n+1) & x(n) \end{pmatrix}
\begin{pmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{pmatrix}
= \begin{pmatrix} x(n) & x(n-1) \end{pmatrix}
A
\]
Obtain the matrix $A$. Then we have $(x(n+1), x(n)) = (x(1), x(0)) A^n$. Obtain $x(8)$ by repeated use of the equation 7 times and the repeated squaring of $A$ three times, and confirm that you have the same result.

Note. Any linear recurrence of the above type is computable in $O(\log n)$ time.

(4) The Euclidean algorithm for greatest common divisors is given as follows:
For $a > b > 0$ such that $a = bq + r$, let $a = r(0)$, $b = r(1)$, $q = q(1)$ and $r = r(2)$. We repeat division as follows:

\[
\begin{align*}
a &= b*q(1) + r(2), & 0 \leq r(2) < b \\
b &= r(2)q(2) + r(3), & 0 \leq r(3) < r(2) \\
&\vdots \\
r(i-1) &= r(i)q(i) + r(i+1), & 0 \leq r(i+1) < r(i) \\
&\vdots \\
r(n) &= r(n)q(n) + r(n+1), & r(n+1) = 0 \\
\text{gcd}(a, b) &= r(n)
\end{align*}
\]

(4.1) Trace this algorithm with $a = 98$ and $b = 77$.

(4.2) Define sequences $c$ and $d$ by

\[
\begin{align*}
c(0) &= 0, & c(1) &= 1, & c(i) &= c(i-2) - q(i-1)c(i-1) \\
d(0) &= 1, & d(1) &= 0, & d(i) &= d(i-2) - q(i-1)d(i-1).
\end{align*}
\]

Then we have $a \cdot d(i) + b \cdot c(i) = r(i)$ for $i=0, ..., n$. By tracing sequences $c$ and $d$, compute $11^{-1} \mod 14$ in the range of $\{1, ..., 13\}$. 